Sign of Gaussian Curvature from Eigen Plane Using Principal Components Analysis

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Abstract

This paper describes a new method to recover the sign of the local Gaussian curvature at each point on the visible surface of a 3-D object. Multiple ($p > 3$) shaded images are acquired under different conditions of illumination. The required information is extracted from a 2-D subspace obtained by applying Principal Components Analysis (PCA) to the p-dimensional space of normalized irradiance measurements.

The number of dimensions is reduced from p to 2 by considering only the first two principal components. The sign of the Gaussian curvature is recovered based on the relative orientation of measurements obtained on a local five point test pattern to those in the 2-D subspace, called the Eigen plane. The method does assume generic diffuse reflectance. The method recovers the sign of Gaussian curvature without assumptions about the light source directions or about the specific functional form of the diffuse surface reflectance.

Multiple ($p > 3$) light sources minimize the effect of shadows by allowing a larger area of visible surface to be analyzed. Results are demonstrated by experiments on synthetic and real data. The results are more accurate and more robust compared to previous approaches.

1 Introduction

Surface curvature is a useful local descriptor of 3-D object shape since it is viewpoint invariant. In computer vision, surface curvature is used for a wide range of tasks including shape recovery, shape modeling, segmentation, object recognition, scene analysis and pose determination.

Local surface curvature can be represented by the values (and associated directions) of the two principal curvatures. Another measure is the Gaussian curvature which is equal to the product of the two principal curvatures. The sign of the Gaussian curvature alone can be useful for specific tasks like segmentation. Several recent papers[1]-[5] describe methods to recover the sign of the Gaussian curvature from $p = 3$ images acquired under different conditions of illumination. This paper proposes a new method to recover the sign of the Gaussian curvature directly from multiple ($p > 3$) images.

The required information is extracted from a 2-D subspace obtained by applying Principal Components Analysis (PCA) to the p-dimensional space of normalized irradiance measurements. The number of dimensions is reduced from p to 2 by considering only the first two principal components. The sign of the Gaussian curvature is recovered based on the relative orientation of measurements obtained on a local five point test pattern to those in the 2-D subspace, called the Eigen plane. The method does assume generic diffuse reflectance. The method recovers the sign of Gaussian curvature without assumptions about the light source directions or about the specific functional form of the diffuse surface reflectance. Results are demonstrated by experiments on synthetic and real data.

2 The Three Light Source Case

In the three light source case, the three irradiance measurements obtained at each pixel are denoted by (E_1, E_2, E_3), where E_1, E_2 and E_3 are considered to define the axes of a 3-D right-handed coordinate system. For a Lambertian surface with constant albedo, Woodham [6] showed that scatter plot of measurements, (E_1, E_2, E_3), define a 6-degree-of-freedom ellipsoid. This ellipsoid does not depend on the shape of the object in view nor on the relative orientation between object and viewer. Angelopoulou [4] showed that scatter plots for a variety of diffuse surfaces with constant albedo, including surfaces with varying degrees of surface roughness, remain ellipsoid-like in that they have positive Gaussian curvature everywhere.

Angelopoulou [4] also showed that the scatter plot for a surface with multiple distinct albedos gives mul-

Proceedings of the International Conference on Pattern Recognition (ICPR'00)
1051-4651/00 $10.00 @ 2000 IEEE

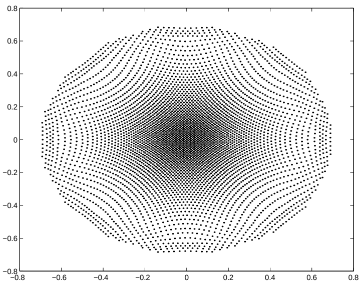

Figure 1. Scatter plot on Eigen Plan from 16 Images for a Lambertian Sphere

tiple distinct ellipsoid-like shapes that differ only in scale.

Following [4], we use normalization to remove the effect of varying albedo. Let $\boldsymbol{E}' = (E_1/\|\boldsymbol{E}\|, E_2/\|\boldsymbol{E}\|, E_3/\|\boldsymbol{E}\|)$. Then, the scatter plot of $\boldsymbol{E}'$ values produces a normalized ellipsoid-like shape in (E_1, E_2, E_3) space. Normalization, as defined here, extends in the obvious way to the p-dimensional case.

3 Sign of the Gaussian Curvature

3.1 Mapping onto the 2-D Eigen Plane

Let $\boldsymbol{\Upsilon}$ be the standard mapping from the unit surface normal at a point on a smooth object to the associated point on the Gaussian sphere. For given conditions of illumination, let $\boldsymbol{\Phi}$ be the mapping from a point on the Gaussian sphere to the p-dimensional space of normalized irradiance measurements. For suitably illuminated points, $\boldsymbol{\Phi}$ is invertible since the p-tuple of image irradiances is different for each different surface normal.

The surface normal itself has only two degrees of freedom. The novel idea is to use Principal Components Analysis (PCA) to reduce the dimensionality of the space of measurements. Each point in the p-dimensional space of the normalized irradiances is mapped into the 2-dimensional subspace by a transformation denoted by $\boldsymbol{\Psi}$. $\boldsymbol{\Psi}$ selects the first two principal components of the original measurements. We call this 2-dimensional subspace the Eigen plane. The essential observation is that the Eigen plane preserves the regularity of points on the Gaussian sphere. This is sufficient to recover the sign of Gaussian curvature, as will be shown.

3.2 Sign of the Transformation

The overall transformation from surface point to Eigen plane is given by $\boldsymbol{\Psi} \circ \boldsymbol{\Phi} \circ \boldsymbol{\Upsilon}$. Let the first and second principal components define the axes of a right-handed 2-D coordinate system for the Eigen plane.

Consider the special case of three light sources and a test object that is a sphere. Define a five point local image template consisting of a center point and top, bottom, left and right neighbours. Label the five points as: 0 for the center point, 1 for the top neighbour, 2, 3 and 4 for the right, bottom and left neighbours respectively (in clockwise order). Given a test object that is a sphere, the corresponding points on the Eigen plane will appear either in clockwise or counter-clockwise order.

All coordinate systems are assumed to be the right-handed coordinate systems. For the three light source case, the points 0 to 4 map into a 3-D space of normalized image irradiances. Further, let's project those normalized image irradiances onto the plane in E_1, E_2, E_3 space through the origin that is perpendicular to the vector (1,1,1). The points 1 to 4 will map onto this plane in a clockwise order if the three light source directions themselves are in counter-clockwise order with respect to the viewing direction. Alternatively, they will map onto this plane in a counter-clockwise order if the three light source directions are in clockwise order. Thus, the preservation or reversal of the clockwise ordering depends explicitly on the ordering of the light source directions with respect to the viewing direction. Without loss of generality, assume that the light sources directions are given in clockwise order with respect to the viewing direction (so that discussion about reversals owing to light source ordering can be avoided).

The transformation $\boldsymbol{\Psi}$ may or may not preserve the clockwise ordering of the points 1 to 4 when they are mapped to the Eigen plane. When $\boldsymbol{\Psi}$ preserves the clockwise ordering, we call it a "positive transformation." When $\boldsymbol{\Psi}$ reverses the clockwise ordering, we call it a "negative transformation."

With p light sources, the ordering of 1 to 4 depends both on the ordering of the light source directions and on $\boldsymbol{\Psi}$. As above, we assume that the light sources directions are given in clockwise order with respect to the viewing direction. The definition of $\boldsymbol{\Psi}$ as a positive or negative transformation remains unchanged.

For a given imaging situation, it is simple to test whether $\boldsymbol{\Psi}$ defines a positive or a negative transformation. Let $\boldsymbol{e_1}, \boldsymbol{e_2}, \cdots, \boldsymbol{e_p}$ be $(1, 0, \cdots, 0)^T, (0, 1, 0, \cdots, 0)^T, \cdots, (0, 0, \cdots, 1)^T$ respectively. Suppose $\boldsymbol{\Psi}$ maps $\boldsymbol{e_1}, \boldsymbol{e_2}, \cdots, \boldsymbol{e_p}$ to $\boldsymbol{e_1'}, \boldsymbol{e_2'}, \cdots, \boldsymbol{e_p'}$ respectively. The distribution of $\boldsymbol{e_1'}, \boldsymbol{e_2'}, \cdots, \boldsymbol{e_p'}$ determines whether $\boldsymbol{\Psi}$ is a positive or negative transformation. With the light sources given in counter-clockwise order, $\boldsymbol{\Psi}$ is a positive transformation if $\boldsymbol{e_1'}, \boldsymbol{e_2'}, \cdots, \boldsymbol{e_p'}$ appear in counter-clockwise order. Conversely, $\boldsymbol{\Psi}$ is a negative transformation if $\boldsymbol{e_1'}, \boldsymbol{e_2'}, \cdots, \boldsymbol{e_p'}$ appear in clockwise order. [ASIDE: if the light sources are given in clockwise order then the sense is simply reversed. That is, $\boldsymbol{\Psi}$ is positive if $\boldsymbol{e_1'}, \boldsymbol{e_2'}, \cdots, \boldsymbol{e_p'}$ appear in clockwise order and negative if they appear in counter-clockwise order]

3.3 Procedure

Table 1 shows how to recover the sign of the Gaussian curvature from the orientation of local test points on the Eigen plane. As before, let the five local points on the image be labeled 0 for the center point, 1 for

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Table 1. How to determine the sign of Gaussian curvature from distribution of local points on the Eigen plane

	$\boldsymbol{\Psi}$	
	positive trans.	negative trans.
clockwise	$G > 0$	$G < 0$
line or a point	$G = 0$	$G = 0$
counterclockwise	$G < 0$	$G > 0$

its upper point, ②, ③ and ④ for the other three points oriented clockwise. Suppose $\boldsymbol{\Psi}$ is a positive transformation. If ① to ④ map onto the Eigen plane in a clockwise manner then $G > 0$. If ① to ④ map onto the Eigen plane in a counter-clockwise manner then $G < 0$. Conversely, suppose $\boldsymbol{\Psi}$ is a negative transformation. If ① to ④ map onto the Eigen plane in a clockwise manner then $G < 0$. If ① to ④ map onto the Eigen plane in a counter-clockwise manner then $G > 0$. Regardless of whether $\boldsymbol{\Psi}$ is positive or negative, if ① to ④ map to a line or a point in the Eigen plane then $G = 0$.

4 Experiments

4.1 Simulated Example

We use a 2-D sinc function (Eq.(1)) as a test surface. Lambertian reflectance is assumed. Eight light source directions are used. One of the eight synthesized images is shown in Figure 2-(a). For the example, $\alpha = 3$. Each image is 256×256 pixels. Gray levels are quantized to 256 values. The albedo (i.e., the constant parameter C in the image irradiance equation $E = C \cos i$) takes on two values, $C = 255$ and $C = 150$ (square areas in Figure 2-(a)). Each image is synthesized under the assumption that the zenith angle of the direction of illumination is 18[deg]. Local four neighboring points are taken two pixels apart around the center pixel.

$$f(x,y) = \alpha \cdot \frac{sinx}{x} \cdot \frac{siny}{y} \qquad (-2\pi < x, y < 2\pi) \quad (1)$$

The estimated results are shown in Figure 2-(b). Figure 2-(c) shows the theoretically calculated result for comparison purposes. The pointwise accuracy for this example is 97.1 %. The results also demonstrate that varying albedo is handled correctly. The 2.9 % error occurs at the boundary between positive and negative Gaussian curvature (i.e., where G is near zero). Values obtained near points of zero Gaussian curvature map to nearby locations in the Eigen plane. This causes the method sometimes to misjudge the orientation of these points leading to errors in the digitized result.

The method successfully estimates the sign of Gaussian curvature even when the light source directions are not widely dispersed. A close arrangement of light source directions results in a high level of correlation between each image. But, PCA is effective in these circumstances leading to robust estimation nevertheless.

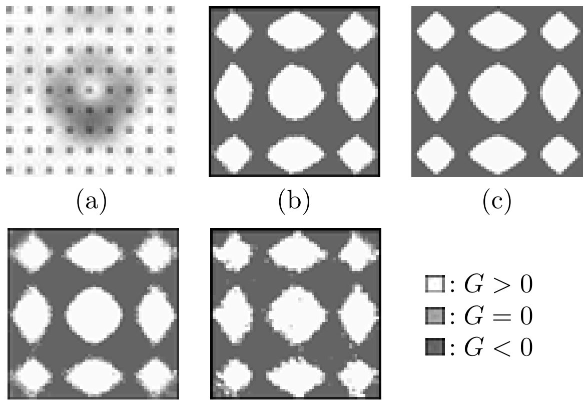


Figure 2. (a) Shaded image (b) Result (c) Theoretical result (d) Result by [3] and (e) Result by [4]

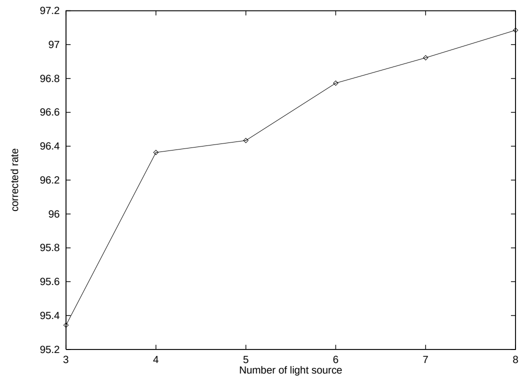


Figure 3. Graph of accuracy versus the number of light sources

We also compare our method to two other methods, [3] and [4]. Both other methods use three light sources. For the comparison, each image also is synthesized under the assumption that the zenith angle of the direction of illumination is 18[deg]. Figure 2-(d) shows a result of method [3] and Figure 2-(e) shows that of [4]. The accuracies are 92.4 % and 91.6 % respectively. Recall that the accuracy for our method using all 8 images (Figure 2-(b)) is 97.1 %.

Figure 3 graphs the accuracy of our method as a function of the number of light sources used. The graph demonstrates the improvement associated with increasing the number of light sources, and therefore the number of images, used.

4.2 Real Examples

A pottery doll is used for experiments on real data. Fifteen light source directions are used. Images are acquired for two different zenith angles of illumination, 8 with a zenith angle of 12.36[deg] and seven with a zenith angle of 16.95[deg].

Three different test poses of the doll are shown in Figure 4. Measurement conditions for each pose are the

(a) doll 1 (b) doll 2 (c) doll 3

Figure 4. Shaded images of test object

(a) doll 1 (b) doll 2 (c) doll 3

Figure 5. Results of dolls

same. Each image is 512×512 pixels. Gray levels are quantized to 256 values. Local four neighboring points are taken in the similar manner as the simulation.

The estimated results are shown in Figure 5. The theoretically correct result is not known. But, qualitatively the estimated sign of Gaussian curvature appears both correct and robust. Figure 5 demonstrates that the result is indeed viewpoint invariant (i.e., independent of pose). The method works for almost the entire visible surface, including points in cast shadow areas such as are found under the jaw and under the hat brim. In general, cast shadows create difficulties for the methods based on only three light sources [3][4].

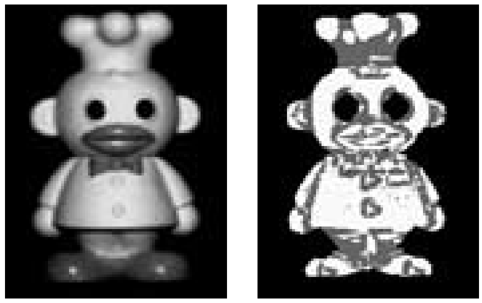
Finally, a second doll (which has varying albedo) is tested. Measurement conditions are identical to those for the examples in Figure 4.

The result is shown in Figure 6-(b). Varying albedo is handled correctly. The estimated sign of the Gaussian curvature appears qualitatively correct except in areas around the lips and boots where the evident specularities deviate from the general diffuse reflectance assumed.

5 Conclusion

This paper described a new method to recover the sign of local Gaussian curvature directly from multiple shaded images. Generic diffuse reflectance is assumed. Principal components analysis is used to reduce a high demensional problem to one of only two dimensions.

The sign of Gaussian curvature is obtained by comparing the relative orientation of five local test points in the image to that of the same points mapped onto the 2-D Eigen plane. This is accomplished without any specific model of diffuse surface reflectance and without specific information about the direction of the light sources.



(a) (b)

Figure 6. (a) Another test object with multiple-albedo and (b) Result

Previous approaches used three light sources. Here, a larger number of light sources (and therefore a larger number of images) are used. Increased accuracy and robustness have been demonstrated, even when the light source directions are not widely dispersed. Spatially varying albedo also is handled correctly.

Specularities do cause the method to fail. (This is true for the other methods cited too [3][4].) Estimating the actual values of surface curvature from multiple images acquired under different conditions of illumination is possible [6] but this does require additional knowledge of the specific reflectance function and measurement conditions involved.

Ackowlegement

Major support for Woodham's research is provided by the IRIS Network of Centres of Excellence and by the Natural Sciences and Engineering Research Council of Canada.

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